



# Prediction of Electromagnetic Wave Propagation in Troposphere Using Parabolic Equation and Two-Dimensional Refractivity

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## Abstract

In this paper, an analysis method of electromagnetic (EM) wave propagation in the troposphere is proposed using the parabolic equation (PE) and the actual two-dimensional modified refractivity (M-unit), that is, range and height dependent M-unit. Discrete Mixed Fourier Transform (DMFT) based PE method is used, and the validation is conducted using AREPS which is developed by the Space and Naval Warfare System Center of US NAVY. The M-units at the National Typhoon Center (NTC) in Jeju-island and meteorological station in Heuksan-island, South Korea are calculated using actual meteorological data. Then, two-dimensional M-unit is estimated using linear interpolation of M-unit at each position. The path loss between NTC and Heuksan-island is analyzed using the three types of M-unit data, Heuksan-island only, NTC only, and linear interpolation. The effect of two-dimensional M-unit on the EM wave propagation when considering long-range problem is discussed.

**Keywords** Parabolic equation method · Two-dimensional modified refractivity · Electromagnetic wave propagation

## 1 Introduction

In long-range radio communications or radar systems, it is important to accurately compute the propagation of electromagnetic (EM) waves in the troposphere. In general, the EM wave propagation in the troposphere is affected by the meteorological and the geographical conditions between the transmitting and receiving antennas [1–4]. Especially, changes in meteorological conditions in accordance with range and height, result in the refractivity variations, and thus the EM wave propagation in the troposphere should be carefully estimated by taking into account these variations of refractivity. For these reasons, many studies have been conducted to analyze the propagation of EM waves

using meteorological data for their countries [5–9]. Previous studies for EM propagation models used the parabolic equation (PE) method [10], the finite element method (FEM) [11], the finite-difference time-domain (FDTD) [12], and the transmission line matrix (TLM) [13]. Among them, the PE method has been widely used because it can consider variations of refractivity and provides efficient solutions for long-range problems [14–17].

However, most previous studies have used the height dependent refractivity only, that is, have not considered the range dependent refractivity. In this paper, an analysis method of the EM wave propagation in the troposphere using the PE method, which can consider the two-dimensional modified refractivity (M-unit) according to both range and height, is proposed. The proposed analysis method employs the PE source code based on Discrete Mixed Fourier Transform (DMFT) [18–20]. Then, the Advanced Refractive Effects Prediction System (AREPS) developed by the Space and Naval Warfare System Center of US NAVY [21], is used to verify the accuracy of our PE simulation results under the simple flat boundary condition. The M-unit data at the transmitting and receiving regions is calculated using the meteorological data from the National Typhoon Center (NTC) in Jeju-island and meteorological station in Heuksan-island, South Korea. The M-units between the two regions are estimated using linear interpolation, which

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results in the two-dimensional M-unit for range and height. Three kinds of M-units such as NTC-only, Heuksan-island only, linearly interpolated, are used to estimate the path loss between NTC and Heuksan-island using the PE. The differences between the three cases are analyzed and the causes of them are discussed.

## 2 Types of Atmospheric Refraction

The troposphere can be characterized by the refractive index  $n$  or refractivity  $N$ , which is dimensionless as follows [22]:

$$n = 1 + 77.6 \times 10^{-6} \frac{P}{T} + 0.373 \frac{e}{T^2} \tag{1}$$

$$N = (n - 1) \times 10^6 \tag{2}$$

where  $P$  is the atmospheric pressure in millibars,  $T$  is the Kelvin temperature, and  $e$  is the water vapor pressure in millibars. The spherical earth can be approximated to a flat earth as shown in Fig. 1 by introducing a fictitious medium where  $N$  is replaced by the modified refractivity  $M$ , which is dimensionless as follows [10]:

$$M = N + \frac{z}{a} 10^6 \tag{3}$$

where  $z$  is the height above surface given in km and  $a$  is the effective radius of the earth, i.e. 6,378 km.

Figure 2 shows the atmospheric conditions and direction of EM wave propagation. In the case of the standard and super refraction, the EM waves are bent to the ground, but they do not reach the ground surface because of the curvature of the earth. On the other hand, if the vertical gradient is greater than 157 M units/km, the EM waves propagate through the atmosphere, not the ground surface, which is called sub-refraction. As a special case, the layer with the vertical gradient of less than 0 is called the duct layer. In the duct layer, the EM waves are bent more to the ground surface than the curvature of the earth and propagate as if they are trapped in the waveguide.

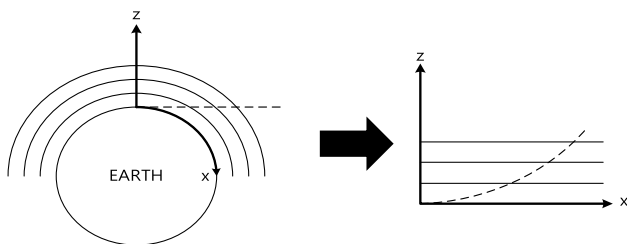


Fig. 1 The spherical and flat earth model

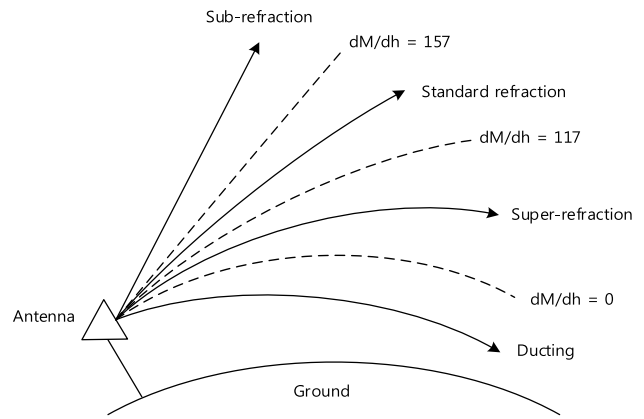


Fig. 2 Types of atmospheric conditions

## 3 Parabolic Equation Method

In the conventional PE method, a large error occurs when the elevation angle is more than 15° since the first order Taylor expansion is used to approximate the Helmholtz equation [10]. In order to reduce the error, the equation is newly approximated as shown in (4), and it works without error up to about 45° [14, 19, 20].

$$\frac{\partial u}{\partial x} - \left[ ik_0^{-1} \left( \sqrt{1 + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} + 1 \right)^{-1} + ik_0(n - 1) \right] u = 0 \tag{4}$$

where  $u$  denotes amplitude of the wave component,  $k_0$  is the wave number in vacuum,  $x$  and  $z$  are range and height respectively. The solution of (4) is given by:

$$u(x + \Delta x, z) = e^{ik_0(m(x,z)-1)\Delta x} \times F^{-1} \left\{ e^{i(\sqrt{k_0^2 - p^2} - k_0)\Delta x} F\{u(x, z)\} \right\} \tag{5}$$

where  $F$  indicates the Fourier transform,  $p = k_0 \sin\theta$  (the propagation angle from the horizontal) is the transform variable, and  $m = n + \frac{z}{a}$  is the modified atmospheric refraction index. As can be seen from (5), the value  $u$  at  $x$  is required to calculate the next step value  $u(x + \Delta x, z)$ . Therefore, to start the PE algorithm, we have to know the value at distance 0 m for the initial value. To define the initial field, the far field pattern of the antenna or the distribution of the aperture is required and converted using the Fourier transform as follows [14]:

$$f(p) = F[a(z)] \tag{6}$$

where  $a$  is the antenna pattern along the height. To consider the finite conductivity boundary, we use image theory to make the field disappear from the under boundary and the condition can be expressed as [14]:

$$U(0, p) = f(p)e^{-ipz} - f^*(-p)e^{ipz} \tag{7}$$

The Gaussian antenna beam pattern is often used as the initial source for the antenna because it can easily adjust the beam width  $\theta_{bw}$  and the elevation angle  $\theta_{elv}$ . The normalized Gaussian antenna beam pattern is as follows [14]:

$$f(p) = \exp \left[ \frac{-p^2 \ln 2}{2k_0^2 \sin^2(\theta_{bw}/2)} \right] \tag{8}$$

The elevation angle variation of the antenna beam can be considered by shifting the Gaussian antenna beam pattern from  $f(p)$  to  $f(p - k_0 \sin \theta_{elv})$

In order to validate the PE simulation result, the path loss was calculated and compared with the AREPS result under the same condition. Figure 3 shows distribution of the path loss calculated by the PE simulation and AREPS. It can be seen that the EM waves are trapped in the duct layer and the results of both simulations are matched well with each other at height below about 1000 m. Above 1000 m, the result has difference since AREPS uses hybrid model [21].

### 4 Path Loss Estimation Using Actual Data

Figure 4 represents actual geographical data from 0 to 188.5 km between NTC and Heuksan-island, obtained from National Geographic Information Institute [23]. For actual M-units along the range and height, two-dimensional M-units are achieved through linear interpolation using the meteorological data at NTC (0 m in distance) and Heuksan-island (188.5 km in distance) from the University of Wyoming as shown in Fig. 5 [24]. The atmospheric conditions are the surface duct from 0 to about 120 km and are standard to the end. The height of the surface duct layer is about 80 m. Therefore, the EM waves from NTC will be trapped from 0 to 80 m in height and will gradually be bent upwards when it is under standard conditions.

Figure 6 represents the path loss distribution calculated by the PE source code using Heuksan-island only, NTC only, and linearly interpolated two-dimensional M-unit. When using Heuksan-island data only, the EM waves are trapped in the surface duct layer as shown in Fig. 6a. On the other hand, if we consider only NTC data which is in the standard atmosphere, the EM waves are bent upward as shown in Fig. 6b. From the previous two results, we can see that calculations using only one side of the data will have a large error compared to the actual results. Therefore, the range and height dependent M-unit, that is, two-dimensional M-unit must be considered. The two-dimensional M-unit between NTC and Heuksan-island is calculated by linear interpolation of the data in each position, and the simulation result using it is shown in Fig. 6c. The path loss distribution

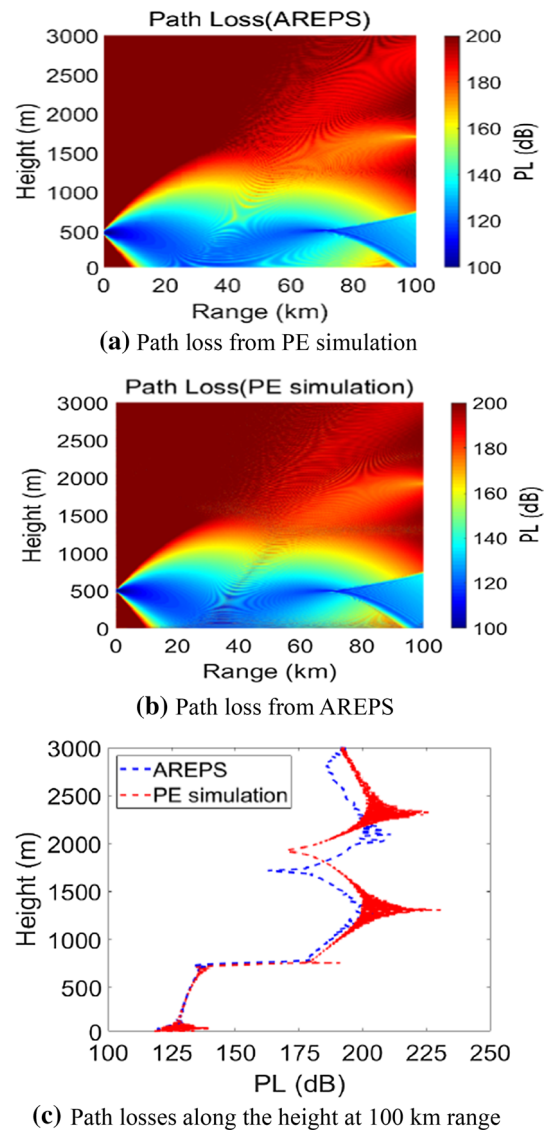


Fig. 3 Validation between the PE simulation and AREPS

in Fig. 6c, from 0 to 40 km in distance, has standard atmospheric characteristics. This is the correct result compared to Fig. 5 in same distance range, and the EM waves in that region is bent upward. On the other hand, from 40 km to the end in distance, the EM waves are trapped by the duct layer up to 120 m. Thus, the path loss is reduced at the end point.

Figure 7 represents three path loss estimation results in terms of the distance when the height of the receiver antenna is fixed at 50 m. Up to about 50 km in range, all results have similar tendency, but the results of three cases are different significantly above 50 km. In the case of the NTC only, the path loss begins to increase at 100 km in distance due to the standard atmosphere. However, since duct layer is considered, both results of the linearly interpolation and Heuksan-island only have relatively low path loss. This shows how important

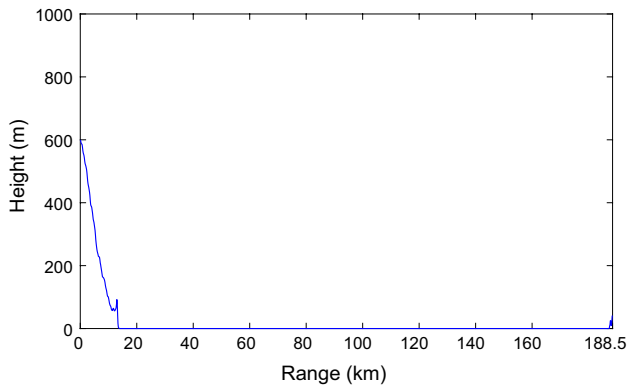
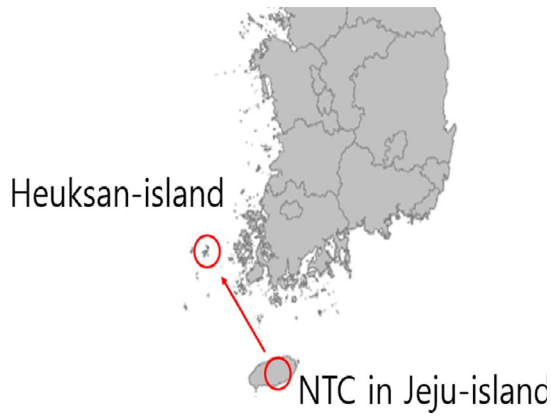


Fig. 4 Geographical data for entire region

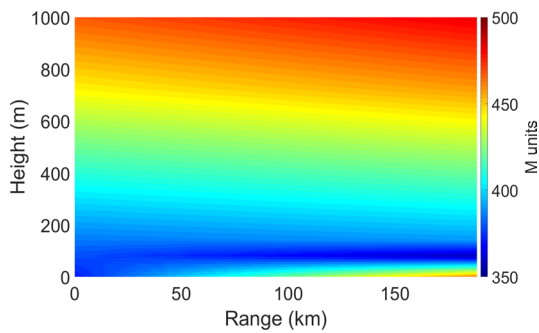
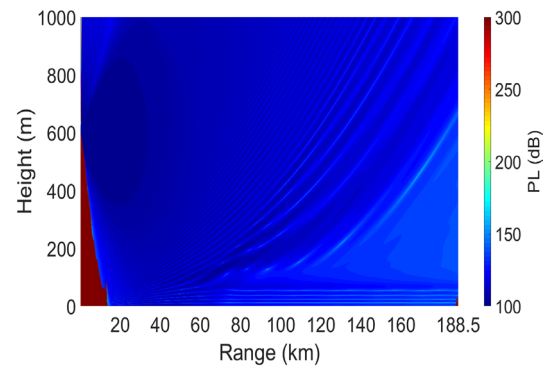


Fig. 5 M-unit data for entire region

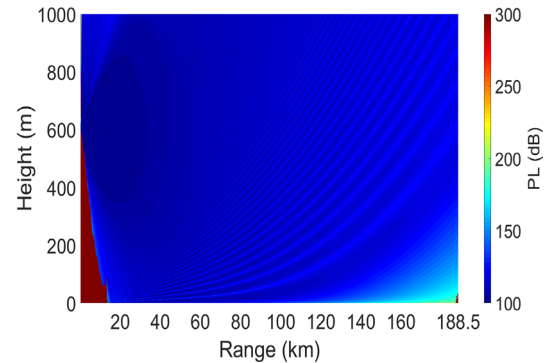
it is to analyze the EM waves propagation considering the two-dimensional M-unit in the long-range problem. The parameters used in the path loss estimation are listed in Table 1.

### 5 Conclusion

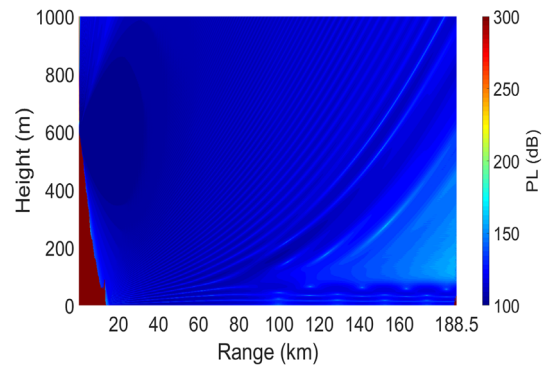
An analysis method of the EM wave propagation in the troposphere using the PE method and the two-dimensional M-unit is proposed. The PE method is based on DMFT, and



(a) Heuksan-island only



(b) NTC only



(c) Linearly interpolated

Fig. 6 Path loss distribution calculated by the PE source code

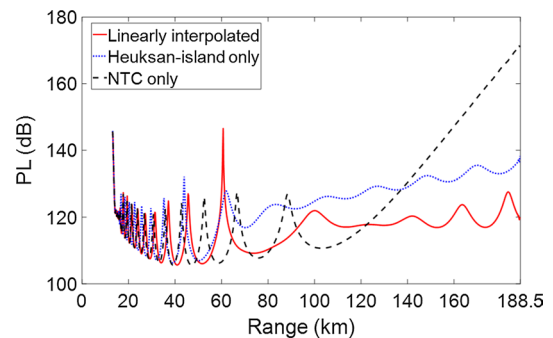


Fig. 7 Path loss estimation along the range

**Table 1** Parameters used in the path loss estimation

| Parameter          | Value                           |
|--------------------|---------------------------------|
| Height             | 0–1000 m                        |
| Range              | 0–188.5 km                      |
| Tx antenna height  | 600 m                           |
| Rx antenna height  | 50 m                            |
| Frequency          | 1 GHz                           |
| Beam width         | 1°                              |
| Elevation angle    | 0.1672°                         |
| Polarization       | Horizontal                      |
| Boundary condition | Perfect electrical conductivity |

the M-units at the NTC and Heuksan-island in South Korea are linearly interpolated for obtaining the two-dimensional M-unit according to both range and height. The path loss between NTC and Heuksan-island is estimated using three M-unit data. Our method can be used to analyze the EM wave propagation considering the inhomogeneity of troposphere with two-dimensional M-unit.

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